Math 53, Discussions 116 and 118 **2022-10-31**

Change of coordinates

Answers

Questions

Question 1. Let D be the region lying between the lines $y = 2x + 1$, $y = 2x + 4$, $y = -3x + 1$, and $y = -3x + 4$. Evaluate

$$\iint_D \frac{y - 2x}{y + 3x}\, \mathrm{d}A$$

Below are brief answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to questions

Question 1. The problem strongly suggests the change of variables $u = y - 2x, v = y + 3x$. The corresponding region in the uv-plane is just the rectangle $1 \le u \le 4, 1 \le v \le 4$.

There are two ways of computing $\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$. One is to first solve for x, y in terms of u, v, obtaining $x = (v-u)/5$ and $y = (3u+2v)/5$. Then we have

$$\left|\det\begin{bmatrix} -1/5 & 1/5 \\ 3/5 & 2/5 \end{bmatrix}\right| = 1/5.$$

Alternatively, one can compute $\left|\frac{\partial(u,v)}{\partial(x,y)}\right|$, which would be

$$\left|\det\begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}\right| = 5$$

and then take its reciprocal. In fact, if you know a little linear algebra, you can see that these two 2×2 matrices are inverses of one another. This is not a coincidence, though we will not discuss it in this course.

After all that, we write down the integral

$$\int_1^4 \int_1^4 \frac{u}{5v}\,du\,dv = \ln 8.$$